

Math 121 Trigonometry

April 2021

1. Question:

Given $\cos(\theta) = -\frac{12}{13}$ and $\tan(\theta) > 0$, find the remaining trig functions at θ .
(exact answers only, no decimal approximations)

Solution

By the Pythagorean theorem, we have:

$$a^2 + b^2 = c^2 \tag{1}$$

where $a = \text{adjacent leg}$, $b = \text{opposite leg}$ and $c = \text{hypotenuse}$. (see figure).

$$(-12)^2 + b^2 = 13^2$$

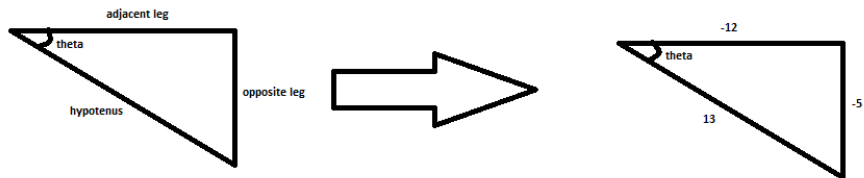
$$b^2 = 13^2 - (-12)^2$$

$$b^2 = 25$$

$$b = \sqrt{25}$$

$$b = \pm 5$$

In this case we take the solution $b = -5$, since $\tan(\theta) > 0$.



Finally, We have:

$$\cos(\theta) = \frac{\text{adjacent leg}}{\text{hypotenuse}} = -\frac{12}{13} \quad (2)$$

$$\cos(\theta) = -\frac{12}{13} \quad (3)$$

$$\sin(\theta) = \frac{\text{opposite leg}}{\text{hypotenuse}} = -\frac{5}{13} \quad (4)$$

$$\sin(\theta) = -\frac{5}{13} \quad (5)$$

$$\tan(\theta) = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{-5}{-12} = \frac{5}{12} \quad (6)$$

$$\tan(\theta) = \frac{5}{12} \quad (7)$$

$$\text{ctg}(\theta) = \frac{\text{adjacent leg}}{\text{opposite leg}} = \frac{-12}{-5} = \frac{12}{5} \quad (8)$$

$$\text{ctg}(\theta) = \frac{12}{5} \quad (9)$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{-\frac{5}{13}} = -\frac{13}{5} \quad (10)$$

$$\csc(\theta) = -\frac{13}{5} \quad (11)$$

$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{-\frac{12}{13}} = -\frac{13}{12} \quad (12)$$

$$\sec(\theta) = -\frac{13}{12} \quad (13)$$

2. Question

Given an angle α that has (8, -6) on its terminal side, find all of the trig functions at α . (exact answers only, no decimal approximations)

3. Solution

In this case we have, $a = \text{adjacent leg} = 8$, $b = \text{opposite leg} = -6$ and $\text{hypotenuse} = 10$

Note: The value of the hypotenuse is found with equation (1)

Finally, We have:

$$\cos(\theta) = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{8}{10} \quad (14)$$

$$\cos(\theta) = \frac{4}{5} \quad (15)$$

$$\sin(\theta) = \frac{\text{opposite leg}}{\text{hypotenuse}} = -\frac{3}{5} \quad (16)$$

$$\sin(\theta) = -\frac{3}{5} \quad (17)$$

$$\tan(\theta) = \frac{\text{opposite leg}}{\text{adjacent leg}} = -\frac{6}{8} \quad (18)$$

$$\tan(\theta) = -\frac{3}{4} \quad (19)$$

$$\text{ctg}(\theta) = \frac{\text{adjacent leg}}{\text{opposite leg}} = -\frac{4}{3} \quad (20)$$

$$\text{ctg}(\theta) = -\frac{4}{3} \quad (21)$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{-\frac{3}{5}} = -\frac{5}{3} \quad (22)$$

$$\csc(\theta) = -\frac{5}{3} \quad (23)$$

$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{\frac{4}{5}} = \frac{5}{4} \quad (24)$$

$$\sec(\theta) = \frac{5}{4} \quad (25)$$

4. Question

Rewrite the expression as a function of a positive acute angle (do not evaluate)

a)

$$\csc(298) \tag{26}$$

5. Solution:

writing positive acute angles:

Q : Quadrant

R : Reference angle

F : Function (sin, cos, tan, csc, ctg, sec) S : Sign (positive or negative)